

Calculation Policy

2015

The following calculation policy has been devised to meet requirements of the National Curriculum 2014 for the teaching and learning of mathematics, and is also designed to give pupils a consistent and smooth progression of learning in calculations across the school. Please note that early learning in number and calculation in Foundation Year follows the 'Development Matters' EYFS document, and this calculation policy is designed to build on progressively from the content and methods established in the Early Years Foundation Stage.

Age Stage Expectations:

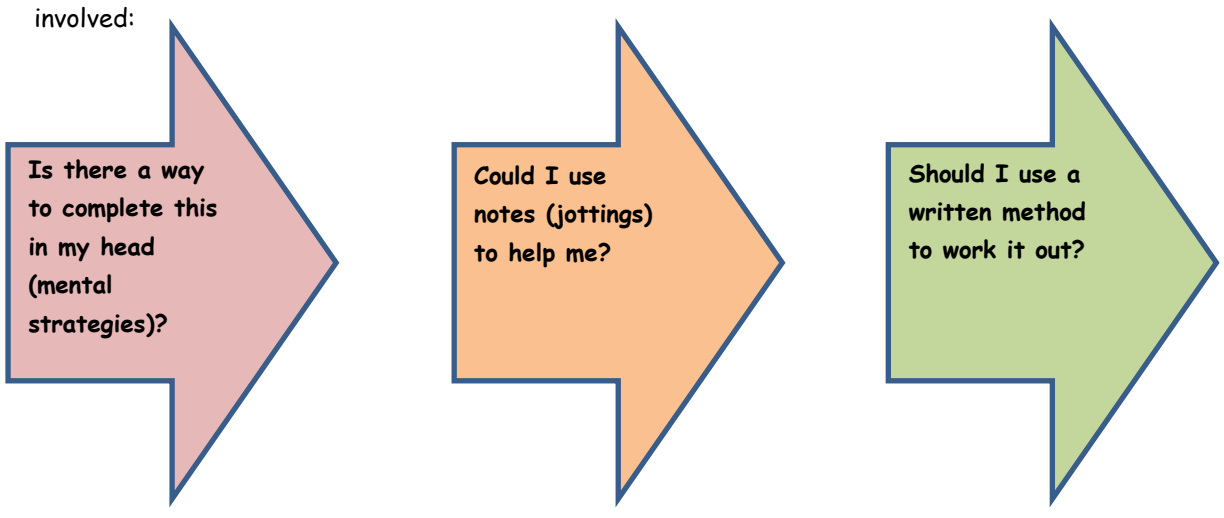
The calculation policy is organised according to age stage expectations as set out in the National Curriculum 2014, **however it is vital that pupils are taught according to the pathway that they are currently working at and children are showing to have 'mastered' a pathway before moving on to the next one.** However, children who are showing to be secure in a skill can be challenged to the next pathway as necessary.

Providing a Context for Calculation:

It is important that any type of calculation is given a real life context or problem solving approach to help build children's understanding of the purpose of calculation, and to help them recognise when to use certain operations and methods when faced with problems. This must be a priority within calculation lessons.

Choosing a Calculation Method:

Children need to be taught and encouraged to use the following processes in deciding what approach they will take to a calculation, to ensure they select the most appropriate method for the numbers involved:



Is there a way to complete this in my head (mental strategies)?

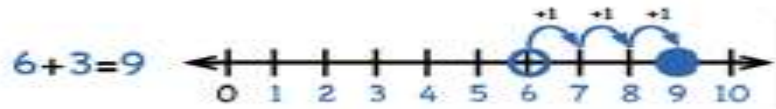
Could I use notes (jottings) to help me?

Should I use a written method to work it out?

Addition Year One

Add with numbers up to 20

Use numbered number lines to add, by counting on in ones. Encourage children to start with the **larger** number and count on.



Children should:

Have access to a wide range of counting equipment, everyday objects, number tracks and number lines, and be shown numbers in different contexts.

Read and write the addition (+) and equals (=) signs within number sentences.

Interpret addition number sentences and solve missing box problems, using concrete objects and number line addition to solve them: $8 + 3 = \square$ $15 + 4 =$

\square $5 + 3 + 1 = \square$ $\square + \square = 6$

This builds on from prior learning of adding by combining two sets of objects into one group (5 cubes and 3 cubes) in Early Years.

$$8 + 5$$



Key skills for addition at Y1:

Read and write numbers to 100 in numerals, incl. 1–20 in words Recall bonds to 10 and 20, and addition facts within 20

Count to and across 100

Count in multiples of 1 2, 5 and 10

Solve simple 1-step problems involving addition, using objects, number lines and pictorial representations.

Year One Addition

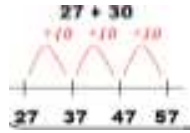
Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line

Addition Year Two

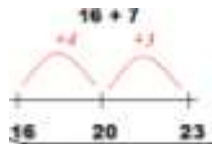
Add with 2-digit numbers

Developing mental fluency with addition and place value involving 2-digit numbers, then establish more formal methods.

Add 2-digit numbers and tens:

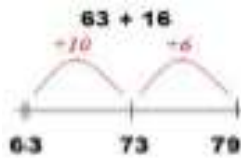


Add 2-digit numbers and units:

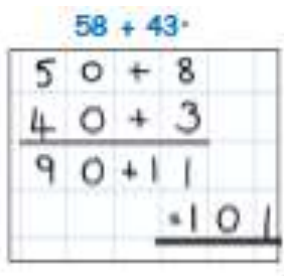


Use empty number lines, concrete equipment, hundred squares etc. to build confidence and fluency in mental addition skills.

Add pairs of two-digit numbers, moving to the partitioned column method when secure adding tens and units.



STEP 1: Only provide examples that do **NOT** cross the tens boundary until they are secure with the method itself.



STEP 2: Once children can add a multiple of ten to a 2-digit number mentally (e.g. 80+11), they are ready for adding pairs of 2-digit numbers that **DO** cross the tens boundary (e.g. 58 + 43).

NEXT STAGE: Children who are **confident** and **accurate** with this stage should move onto the expanded addition methods with 2 and 3-digit

To support understanding, pupils may physically make and carry out the calculation with Diennes Base 10 apparatus or place value counters, then compare their practical version to the written form, to help them to build an understanding of it.

Key skills for addition at Y2:

- Add a 2-digit number and ones (e.g. 27 + 6)
- Add a 2-digit number and tens (e.g. 23 + 40)
- Add pairs of 2-digit numbers (e.g. 35 + 47)
- Add three single-digit numbers (e.g. 5 + 9 + 7)
- Show that adding can be done in any order (the commutative law)
- Recall bonds to 20 and bonds of tens to 100 (30 + 70 etc.)
- Count in steps of 2, 3 and 5 and count in tens from any number
- Understand the place value of 2-digit numbers (tens and ones)
- Compare and order numbers to 100 using < > and = signs
- Read and write numbers to at least 100 in numerals and words
- Solve problems with addition, using concrete objects, pictorial representations, involving numbers quantities and measures, and applying mental and written methods.

Year Two Addition

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary

Addition Year Three

Add numbers with up to 3-digits.

Introduce the **expanded column addition** method:

In order to carry out this method of addition:

- Children need to recognise the value of the hundreds, tens and units without recording the partitioning.
- Pupils need to be able to add in columns.

$$\begin{array}{r} 236 \\ + 73 \\ \hline 309 \end{array}$$

Add the **units** first, in preparation for the compact method.

Move to the compact **column addition** method, with carrying:

Children who are very secure and confident with 3-digit expanded column addition should be moved onto the **compact column addition** method, being introduced to carrying for the first time. Compare the expanded method to the compact column method to develop an understanding of the process and the reduced number of steps involved.

236

+ 73

309

1

Add the **units** first

'Carry' numbers **underneath** the bottom line.

Remind pupils the actual value is '**three tens add seven tens**', not '**three add seven**', which equals **ten tens**.

Key skills for addition at Y3:

Read and write numbers to 1000 in numerals and words.

Add 2-digit numbers mentally, incl. those exceeding 100.

Add a three-digit number and ones mentally (175 + 8)

Add a three-digit number and tens mentally (249 + 50)

Add a three-digit number and hundreds mentally (381 + 400)

Estimate answers to calculations, using inverse to check answers

Solve problems, including missing number problems, using

Number facts, place value, and more complex addition.

Recognise place value of each digit in 3-digit numbers

(hundreds, tens, ones)

Continue to practise a wide range of mental addition strategies.

i.e. Number Bonds, adding the nearest multiple of 10, 100, 100

and adjusting, using near doubles, partitioning and recombining.

Year Three Addition Key Vocabulary

add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, **hundreds boundary**, **increase**, **vertical**, **'carry'**, **expanded**, **compact**

Addition Year Four

Add numbers with up to 4 digits

Move from expanded addition to the compact column method, **adding units first**, and 'carrying' numbers **underneath** the calculation. Also include money and measures contexts.

e.g. $3517 + 396 = 3913$

	3	5	1	7
+		3	9	6
<hr/>				
	3	9	1	3

'Carry' numbers **underneath** the bottom line.

Add the **units** first

Reinforce correct place value by reminding them the actual value is **5 hundreds** add **3 hundreds**, not **5 add 3**, for example.

Use and apply this method to money and measurement values.

Introduce the **compact column addition** method by asking children to add the two given numbers together using the method that they are familiar with (**expanded column addition—see Y3**). Teacher models the compact method with carrying, asking children to discuss similarities and differences and establish how it is carried out. Make sure that pupils draw a line underneath the final calculation.

Key skills for addition at Y4:

- Select most appropriate method: mental, jottings or written and explain why
- Recognise the place value of each digit in a four-digit number
- Round any number to the nearest 10, 100 or 1000
- Estimate and use inverse operations to check answers
- Solve 2-step problems in context, deciding which operations and methods to use and why
- Find 1000 more or less than a given number
- Continue to practise a wide range of mental addition strategies.
ie. Number Bonds, add the nearest multiple of 10, 100, 1000 and adjust, use near doubles, partitioning and recombining
- Solve 2-step problems in contexts, deciding which operations and methods to use and why
- Estimate and use inverse operations to check answers to a calculation.

Year Four Addition

Key Vocabulary

add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, vertical, 'carry', expanded, compact, **thousands, hundreds, digits, inverse**

Addition Year Five

Add numbers with more than 4 digits including money, measures and decimals with different numbers of decimal places.

A handwritten addition problem on a grid background. The first row is £23.59, the second row is + £7.55, and the third row is the result £31.14. A blue arrow points from the text box on the right to the decimal point in the first row.

The decimal point should be aligned in the same way as the other place value columns, and must be in the same column in the answer. (Keep reminding pupils about the line under the final calculation).

Numbers should exceed 4 digits.

Pupils should be able to add more than two values, carefully aligning place value columns.

A handwritten addition problem on a grid background. The first row is 19.01, the second row is 3.65, the third row is + 0.7, and the fourth row is the result 23.36. A blue arrow points from the text box on the right to the decimal point in the second row.

Say '6 tenths
add 7 tenths'
to reinforce
place value.

Empty decimal places can
be filled with zero to show
the place value in each
column.

CHILDREN SHOULD:

Understand the place value of **tenths** and **hundredths** and use this to align numbers with different numbers of decimal places.

Key skills for addition at Y5:

- Add numbers mentally with increasingly large numbers, using and practising a range of mental strategies. i.e. Add the nearest multiple of 10, 100, 100 and adjust;
- use near doubles, inverse, partitioning and re-combining; using number bonds
- Use rounding to check answers and accuracy
- Solve multi-step problems in contexts, deciding which operations and methods to use and why
- Read, write, order and compare numbers to at least 1 million and determine the value of each digit
- Round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000

Year Five Addition

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, 'carry', expanded, compact, vertical, thousands, hundreds, digits, inverse & **decimal places, decimal point, tenths, hundredths, thousandths**

Addition Year Six

Add several numbers of increasing complexity

Adding several numbers with different numbers of decimal places (including money and measures): Tenths, hundredths and thousandths should be correctly aligned, with the decimal point lined up vertically including in the answer row. Zeros could be added into any empty decimal places, to show there is no value to add.

	2	3	.	3	6	1
		9	.	0	8	0
	5	9	.	7	7	0
+		1	.	3	0	0
<hr/>						
	9	3	.	5	1	1
	2	1		2		

Empty decimal places can be filled with zero to show the place value in each column.

Adding several numbers with more than 4 digits.

	8	1	,	0	5	9
				3	6	6
				1	5	3
				2	0	5
+				5	5	1
<hr/>						
	1	2	0	5	7	9
				1	1	1

Key skills for addition at Y6:

- Perform mental calculations, including with mixed operations and large numbers, using and practising a range of mental strategies
- Solve multi-step problems in context, deciding which operations and methods to use and why Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy
- Read, write, order and compare numbers up to 10 million and determine the value of each digit.
- Round any whole number to a required degree of accuracy
- Pupils understand how to add mentally with larger numbers and calculations of increasing complexity.

Year Six Addition

Key Vocabulary:

add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, 'carry', expanded, compact, vertical, thousands, hundreds, digits, inverse, decimal places, decimal point, tenths, hundredths, thousandths

Subtraction Year One

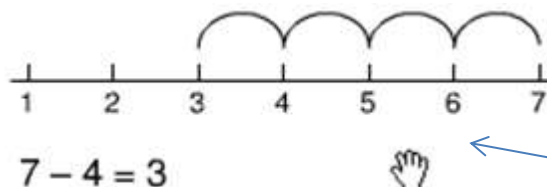
Read, write and interpret number sentence with - and = signs.

Subtract from numbers up to 20

Children consolidate understanding of subtraction practically, showing subtraction on bead strings, using cubes etc. and in familiar contexts, and are introduced to more formal recording using number lines as below:

SUBTRACT BY TAKING AWAY

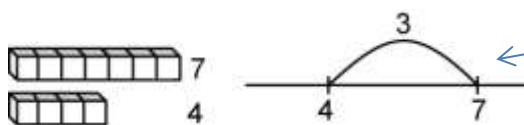
Count back in ones on a numbered number line to take away, with numbers up to 20



Model subtraction using hundred squares and numbered number lines/tracks and practically.

FIND THE 'DIFFERENCE BETWEEN'

This will be introduced practically with the language 'find the difference between' and 'how many more?' In a range of familiar contexts.



The difference between 7 and 4 is 3.

Seven is 3 more than four

I am 2 years older than my sister

Mental subtraction

Children should start recalling subtraction facts up to **and within** 10 and 20, and should be able to subtract zero.

Year One Subtraction

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is_?

Key skills for subtraction at Y1:

- Given a number, say **one more or one less**
- Count to and over 100, **forward and back**, from any number
- Represent and use **subtraction facts to 20 and within 20**
- Subtract with **one-digit and two-digit** numbers to 20, including zero
- Solve one-step problems that involve addition and subtraction, using concrete objects (i.e. bead string, objects, cubes) and pictures, and missing number problems
- Read and write numbers from 0 to 20 in numerals and words.

Subtraction Year Two

Subtract with 2-digit numbers

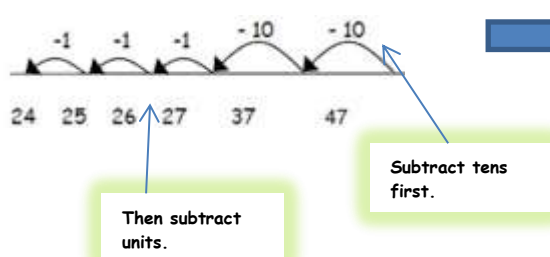
Subtract on a number line by counting back, aiming to develop mental subtraction skills.

This strategy will be used for:

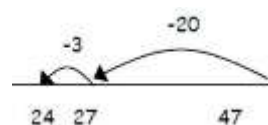
- 2-digit numbers subtract units (by taking away / counting back) e.g. $36-7$
- 2-digit numbers subtract tens (by taking away / counting back) e.g. $48-30$
- Subtracting pairs of 2-digit numbers (see below)

Subtracting pairs of 2-digit numbers on a number line:

$47 - 23 = 24$ Partition the second number and subtract it in tens and units, as below:

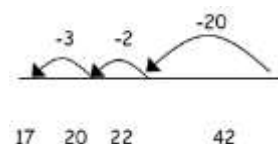


Move towards more efficient jumps back, as below:



Combine methods with use of a hundred square to reinforce understanding of number value and order.

Teaching children to **bridge through ten** can help them to become more efficient, for example $42-25$:

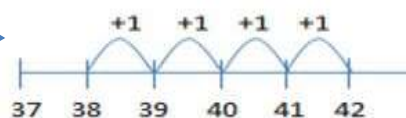


Mental strategy - subtract numbers close together by counting on:

Many mental strategies are taught. Children are taught to recognise that when numbers are close together, it is more efficient to **count on** the difference. They need to be clear about the relationship between addition and subtraction.

Start with the smaller number and count on to the largest.

$$42 - 38 = 4$$



Key skills for subtraction at Y2:

- Recognise the place value of each digit in a two-digit number.
- Recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100.
- Subtract using concrete objects, pictorial representations, 100 squares and mentally, including: a two-digit number and ones, a two-digit number and tens, and two two-digit numbers
- Show that subtraction of one number from another cannot be done in any order
- Recognise and use inverse relationship between addition and subtraction, using this to check calculations and missing number problems
- Solve simple addition and subtraction problems including measures, using concrete objects, pictorial representation, and also applying their increasing knowledge of mental and written methods
- Read and write numbers to at least 100 in numerals and in words.

Year Two Subtraction

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is_? , **difference**, **count on**, **strategy**, **partition**, **tens**, **units**

Subtraction Year Three

Subtracting with 2 and 3-digit numbers. Introduce **partitioned column subtraction** method.

STEP 1: introduce this method with examples where no exchanging is required.

$$89 - 35 = 54$$

$$\begin{array}{r} 80 + 9 \\ - 30 + 5 \\ \hline 50 + 4 \end{array}$$

STEP 2: introduce 'exchanging' through practical subtraction. Make the larger number with Base 10, then subtract 47 from it.

$$72 - 47$$



$$\begin{array}{r} 60 \\ 70 + 2 \\ - 40 + 7 \\ \hline 20 + 5 = 25 \end{array}$$

When learning to 'exchange', explore partitioning in different ways so that pupils understand that when you exchange, the **VALUE** is the same i.e. $72 = 70 + 2 = 60 + 12 = 50 + 22$ etc. Emphasise that the **value** hasn't changed, we have just partitioned it in a different way.

STEP 3: Once pupils are secure with the understanding of "exchanging", they can use the partitioned column method to subtract any 2 and 3-digit numbers.

$$\begin{array}{r} 238 - 146 = 92 \\ \hline 200 + 30 + 8 \\ - 100 + 40 + 6 \\ \hline 100 + 90 + 2 \end{array}$$

Subtracting money: partition into e.g. £1 + 30p + 8p

Counting on as a mental strategy for subtraction:

Continue to reinforce counting **on** as a strategy for **close-together numbers** (e.g. 121–118), and also for numbers that are nearly multiples of 10, 100, 1000 or £'s, which make it easier to count on (e.g. 102–89, 131–79, or calculating change from £1 etc.).

Start at the smaller number and count on in **tens** first, then count on in units to find the rest of the difference:



Key skills for subtraction at Y3:

- Subtract mentally at: **3-digit number and ones**, **3-digit number and tens**, **3-digit number and hundreds** Estimate answers and use inverse operations to check
- Solve problems, including missing number problems Find 10 or 100 more or less than a given number
- Recognise the place value of each digit in a 3-digit number
- Counting up differences as a mental strategy when numbers are close together or near multiples of 10 (see examples above)
- Read and write numbers up to 1000 in numerals and words
- Practise mental subtraction strategies, such as subtracting near multiples of 10 and adjusting

Year Three Subtraction

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is_?, difference, count on, strategy, partition, tens, units **exchange**, **decrease**, **hundreds**, **value**, **digit**

Subtraction Year Four

Subtract with up to 4-digit numbers

Partitioned column subtraction with exchanging (decomposition):

$$\begin{array}{r}
 2754 - 1562 = 1192 \\
 \hline
 2000 + \cancel{700}^{600} + 50 + 4 \\
 - 1000 + 500 + 60 + 2 \\
 \hline
 1000 + 100 + 90 + 2
 \end{array}$$

As introduced in Y3, but moving towards more complex numbers and values. Use **place value counters** to reinforce 'exchanging'.

Subtracting money: partition into £1 + 30 + 5 for example

Compact column subtraction

To introduce the compact method, ask children to perform a subtraction calculation with the familiar partitioned column subtraction then display the compact version for the calculation they have done. Ask pupils to consider how it relates to the method they know, what is similar and what is different, to develop an understanding of it.

Give plenty of opportunities to apply this to money and measures.

$$\begin{array}{r}
 2754 \\
 - 1562 \\
 \hline
 1192
 \end{array}$$

Always encourage children to consider the best method for the numbers involved—mental, counting on, counting back or written method.

Mental strategies

A variety of mental strategies must be taught and practised, including counting on to find the difference where numbers are closer together, or where it is easier to count on.

Key skills for subtraction at Year 4:

Subtract by counting on where numbers are close together or they are near to multiples of 10, 100 etc.

Children select the most appropriate and efficient methods for given subtraction calculations

Estimate and use inverse operations to check answers

Solve addition and subtraction 2-step problems, choosing which operations and methods to use and why

Solve simple measure and money problems involving fractions and decimals to two decimal places

Find 1000 more or less than a given number

Count backwards through zero, including negative numbers

Recognise place value of each digit in a 4-digit number Round any number to the nearest 10, 100 or 1000

Solve number and practical problems that involve the above, with increasingly large positive numbers.

Year Four Subtraction

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance

between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is __?, difference, count on, strategy, partition, tens, units exchange, decrease, hundreds, value, digit, **inverse**

Subtraction Year Five

Subtract with at least 4-digit numbers including money, measures, decimals.

Compact column subtraction (with 'exchanging').

Subtracting with larger integers.

$$\begin{array}{r} \cancel{2}^{\text{h}} \cancel{1}^{\text{t}} \cancel{0}^{\text{h}} \cancel{8}^{\text{t}} \text{'6} \\ - \quad \quad 2 \text{ } 1 \text{ } 2 \text{ } 8 \\ \hline 2 \text{ } 8 \text{ } , \text{ } 9 \text{ } 2 \text{ } 8 \end{array}$$

Children who are still not secure with number facts and place value will need to remain on the partitioned column method until ready for the compact method.

Subtract with decimal values, including mixtures of integers and decimals, aligning the decimal point.

$$\begin{array}{r} \cancel{7}^{\text{h}} \cancel{1}^{\text{t}} \cancel{6}^{\text{h}} \cancel{9}^{\text{t}} \cdot \text{'0} \\ - \quad \quad 3 \text{ } 7 \text{ } 2 \cdot 5 \\ \hline 6 \text{ } 7 \text{ } 9 \text{ } 6 \cdot 5 \end{array}$$

Add a zero in any empty decimal places to aid understanding of what to subtract in that column.

Create lots of opportunities for subtracting and finding differences with money and measures.

Key skills for subtraction at Y5:

Subtract numbers mentally with increasingly large numbers

Use rounding and estimation to check answers to calculations and determine, in a range of contexts, levels of accuracy

Solve addition and subtraction multi-step problems in context, deciding which operations and methods to use and why

Read, write, order and compare numbers to at least 1 million and determine the value of each digit

Count forwards or backwards in steps of powers of 10 for any given number up to 1 million

Interpret negative numbers in context, counting forwards and backwards with positive and negative integers through 0

Round any number up to 1 million to the nearest 10, 100, 1000, 10 000 and 100 000.

Year Five Subtraction

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance

between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is_?, difference, count on, strategy, partition, tens, units, exchange, decrease, hundreds, value, digit, inverse, **tenths**, **hundredths**, **decimal point**, **decimal**

Subtraction Year Six

Subtracting with increasingly large and more complex numbers and decimal values.

$$\begin{array}{r}
 \cancel{9}^4 \cancel{5}^9 \cancel{10}^9, 699 \\
 - 89,949 \\
 \hline
 60,750
 \end{array}$$

Using the compact column method to subtract more complex integers.

$$\begin{array}{r}
 \cancel{10}^5 \cdot \cancel{4}^1 9 \text{ kg} \\
 - 36 \cdot 08 \text{ kg} \\
 \hline
 69 \cdot 339 \text{ kg}
 \end{array}$$

Using the compact column method to subtract money and measures, including decimals with different numbers of decimal places.

Empty decimal places can be filled with **zero** to show the place value in each column.

Pupils should be able to apply their knowledge of a range of mental strategies, mental recall skills, and informal and formal written methods when selecting **the most appropriate method** to work out subtraction problems.

Key skills for subtraction at Y6:

- Solve addition and subtraction multi-step problems in context, deciding which operations and methods to use and why.
- Read, write, order and compare numbers up to 10 million and determine the value of each digit Round any whole number to a required degree of accuracy
- Use negative numbers in context, and calculate intervals across zero.
- Children need to utilise and consider a range of mental subtraction strategies, jottings and written methods before choosing how to calculate.


Year Six Subtraction

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is _? difference, count on, strategy, partition, tens, units exchange, decrease, hundreds, value, digit, inverse, tenths, hundredths, decimal point, decimal

Multiplication Year One

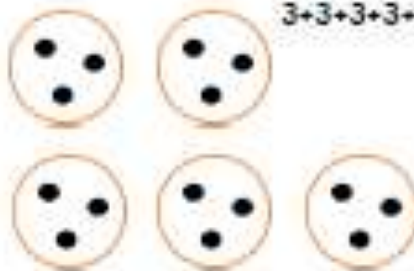
Multiply with concrete objects, arrays and pictorial representations.

How many legs will 3 teddies have?



$2 + 2 + 2 = 6$

There are 3 sweets in one bag.
How many sweets are in 5 bags altogether?



$3+3+3+3+3 = 15$

Give children experience of counting equal group of objects in 2s, 5s and 10s.

Present practical problem solving activities involving counting equal sets or groups, as above.

Key skills for multiplication at Year 1:

- Count in multiples of 2, 5 and 10
- Solve one-step problems involving multiplication, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher
- Make connections between arrays, number patterns, and counting in twos, fives and tens
- Begin to understand doubling using concrete objects and pictorial representations.

Year One Multiplication

Key vocabulary:

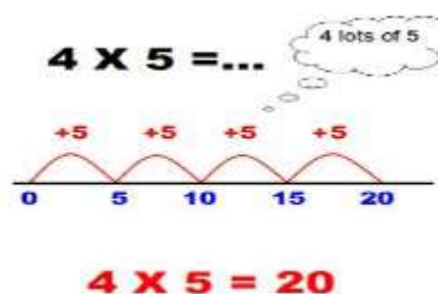
groups of , lots of , times, array, altogether, multiply, count

Multiplication Year Two

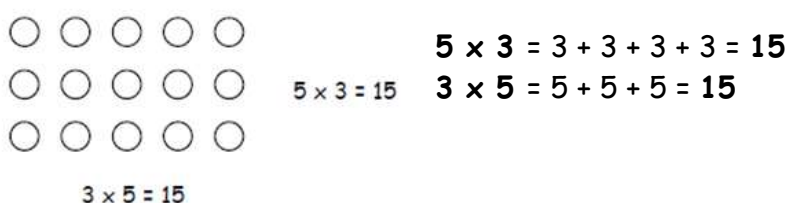
Multiply using arrays and repeated addition (using at least 2s, 5s and 10s)

Use repeated addition on a number line:

Starting from zero, make equal jumps up on a number line to work out multiplication facts and write multiplication statements using \times and $=$ signs.

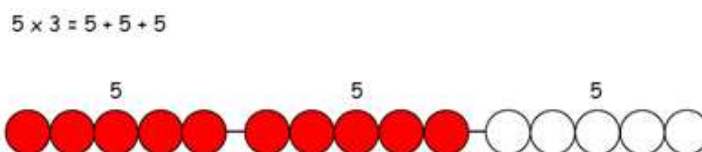


Use arrays



Use arrays to help teach children to understand the commutative law of multiplication, and give examples such as $3 \times \underline{\quad} = 6$.

Use practical apparatus



Use mental recall:

Children should begin to **recall multiplication facts for 2, 5 and 10** times tables through practice in counting and understanding of the operation.

Key skills for multiplication at Y2:

Count in steps of 2, 3 and 5 from zero, and in 10s from any number

Recall and use multiplication facts from the **2, 5 and 10** multiplication tables, including recognising odds and evens

Write and calculate number statements **using the \times and $=$ signs** Show that multiplication can be done in any order (commutative)

Solve a range of problems involving multiplication, using concrete objects, arrays, repeated addition, mental methods, and multiplication facts

Pupils use a variety of language to discuss and describe multiplication.

Year Two Multiplication

Key vocabulary:

groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times...

Multiplication Year Three

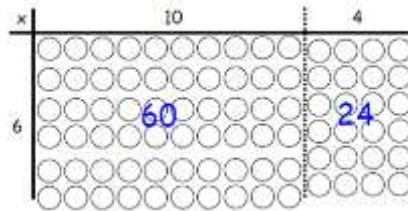
Multiply 2-digits by a single digit number

Introduce the grid method for multiplying 2-digit by single-digits:

Eg. $23 \times 8 = 184$

Link the layout of the grid to an array initially

X	20	3
8	160	24



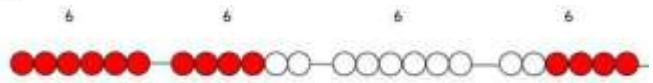
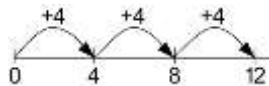
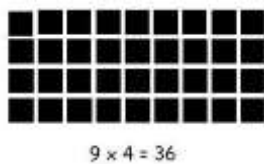
Introduce the grid method with children physically making an array to represent the calculation (e.g. make 8 lots of 23 with 10s and 1s place value counters), then translate this to grid method format.

To do this, children must be able to:

Partition numbers into tens and units

Multiply multiples of ten by a single digit (e.g. 20×4) using their knowledge of multiplication facts and place value

Recall and work out multiplication facts in the **2, 3, 4, 5, 8 and 10** times tables. Work out multiplication facts not known by repeated addition or other taught mental strategies (e.g. by commutative law, working out near multiples and adjusting, using doubling etc.) Strategies to support this are: repeated addition using a number line, bead bars and arrays:



Key skills for multiplication Year 3:

Recall and use multiplication facts for the **2, 3, 4, 5, 8 and 10** multiplication tables, and multiply multiples of 10

Write and calculate number statements using the multiplication tables they know, including **2-digit x single-digit**, drawing upon mental methods, and progressing to reliable written methods

Solve multiplication problems, including missing number problems

Develop mental strategies using commutativity (e.g. $4 \times 12 \times 5 = 4 \times 5 \times 12 = 20 \times 12 = 240$)

Solve simple problems in contexts, deciding which operations and methods to use

Develop efficient mental methods to solve a range of problems e.g using commutativity

($4 \times 12 \times 5 = 4 \times 5 \times 12 = 20 \times 12 = 240$) and for missing number problems $\times 5 = 20$, $3 \times = 18$, $\times = 32$

Year Three Multiplication

Key vocabulary:

groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times, _times, as big as, once, twice, three times..., **partition, grid method, multiple, product, tens, units, value**

Multiplication Year Four

Multiply 2 and 3-digits by a single digit, using all multiplication tables up to 12×12

Developing the grid method:

Eg. $136 \times 5 = 680$

X	100	30	6
5	500	150	30

$$\begin{array}{r} 500 \\ 150 \\ + 30 \\ \hline 680 \end{array}$$

Encourage column addition to add accurately.

Move onto **short multiplication** (see Y5) if and when children are confident and accurate multiplying 2 and 3-digit numbers by a single digit this way, **and** are already confident in 'carrying' for written addition.

Children should be able to:

- **Approximate before they calculate**, and make this a regular part of their calculating, going back to the approximation to check the reasonableness of their answer. e.g: 346×9 is approximately $350 \times 10 = 3500$
- Record an approximation to check the final answer against.
- Multiply multiples of ten and one hundred by a single-digit, using their multiplication table knowledge
- Recall all times tables **up to 12×12** .

Key skills for multiplication at Year 4:

- Count in multiples of 6, 7, 9, 25 and 1000
- Recall multiplication facts for **all multiplication tables up to 12×12** Recognise place value of digits in up to 4-digit numbers
- Use place value, known facts and derived facts to multiply mentally, e.g. multiply by 1, 10, 100, by 0, or to
- multiply 3 numbers
- Use **commutativity** and other strategies mentally $3 \times 6 = 6 \times 3$, $2 \times 6 \times 5 = 10 \times 6$, $39 \times 7 = 30 \times 7 + 9 \times 7$ Solve problems with increasingly complex multiplication in a range of contexts
- Count in multiples of 6, 7, 9, 25 and 1000
- Recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones)

Year Four Multiplication

Key vocabulary:

groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, array, column, row, commutative, groups of, sets of, lots of, equal groups, times, multiply, times as big as, once, twice, three times... partition, grid method, total, multiple, product, sets of, **inverse**

Multiplication Year Five

Multiply up to 4-digits by 1 or 2 digits.

Introducing column multiplication

Introduce by comparing a grid method calculation to a short multiplication method, to see how the steps are related, but notice how there are less steps involved in the column method. Children need to be taught to approximate first, e.g. for 72×38 , they will use rounding: 72×38 is approximately $70 \times 40 = 2800$, and use the approximation to check the reasonableness of their answer against.

Short multiplication for multiplying by a single digit

x	300	20	7
4	1200	80	28



	3	2	7	
x			4	
	1	3	0	8
		1	2	

Pupils could be asked to work out a given calculation using the grid, and then compare it to their column method. What are the similarities and differences? Unpick the steps and show how it reduces the steps.

Introduce long multiplication for multiplying by 2 digits

	10	8
10	100	80
3	30	24



		1	8
x		1	3
		5	4
	1	8	0
	2	3	4

18×3 on the 1st row ($8 \times 3 = 24$, carrying the 2 for twenty, then 1×3).
 18×10 on the 2nd row. Put a zero in units first, then say 8×1 , and 1×1 .

Moving towards more complex numbers:

	1	2	3	4
x			1	6
	7	4	0	4
	1	2	3	4
	1	9	7	4

1234×6
 (1234×10)

	3	6	5	2
x				8
	2	9	2	1
	5	4		

Key skills for multiplication at Y5:

- Identify multiples and factors, using knowledge of **multiplication tables to 12x12**.
- Solve problems where larger numbers are decomposed into their factors
- Multiply and divide integers and decimals by 10, 100 and 1000 Recognise and use square and cube numbers and their notation
- Solve problems involving combinations of operations, choosing and using calculations and methods appropriately.

Year Five Multiplication

Key vocabulary:

groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, _times as big as, once, twice, three times..., partition, grid method, total, multiple, product, inverse, **square**, **factor**, **integer**, **decimal**, **short/long multiplication**, 'carry'

Multiplication Year Six

Short and long multiplication as in Year 5, and single digit.

	3	.	1	9
x	8			
<hr/>				
2	5	.	5	2
	1		7	

This works well for multiplying money (£.p) and

Line up the decimal points in the question and the answer.

Remind children that the single digit belongs in the units column.

Children will be able to:

- Use rounding and place value to make approximations before calculating and use these to check answers against.
- Use **short multiplication** (see Y5) to multiply numbers with **more than 4-digits by a single digit**; to multiply money and measures, and to **multiply decimals with up to 2d.p. by a single digit**.
- Use **long multiplication** (see Y5) to multiply numbers with **at least 4 digits by a 2-digit number**.

Key skills for multiplication at Y6:

- Recall multiplication facts for all times tables up to **12 x 12 (as Y4 and Y5)**
- Multiply multi-digit numbers, up to 4-digit x 2-digit using long multiplication
- Perform mental calculations with mixed operations and large numbers
- Solve multi-step problems in a range of contexts, choosing appropriate combinations of operations and methods
- Estimate answers using round and approximation and determine levels of accuracy
- Round any integer to a required degree of accuracy.

Year Six Multiplication

Key vocabulary:

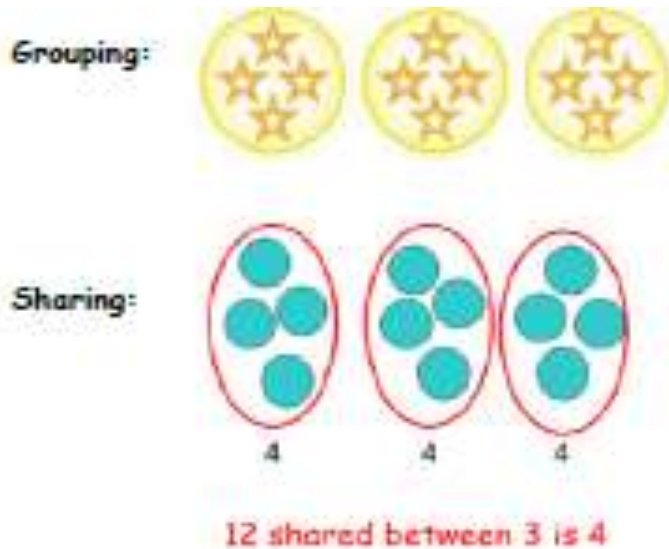
groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, array, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times... partition, grid method, total, multiple, product, inverse, square, factor, integer, decimal, short / long multiplication, 'carry', tenths, hundredths, decimal

Division Year One

Group and share small quantities

Use objects, diagrams and pictorial representations to solve problems involving **both** grouping and sharing.

How many groups of 4 can be made with 12 stars? = 3



Example division problem in a familiar context:

There are 6 pupils on this table and there are 18 pieces of fruit to share between us. If we share them equally, how many will we each get? Can they work it out and give a division statement... ?

"18 shared between 6 people gives you 3 each."

Pupils should:

- use lots of practical apparatus, arrays and picture representations
- be taught to understand the difference between **grouping** objects (How many groups of 2 can you make?) and **sharing** (Share these sweets between 2 people)
- be able to count in multiples of 2s, 5s and 10s
- find **half** of a group of objects by sharing into 2 equal groups.

Key number skills needed for division at Year 1:

- Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations arrays with the support of the teacher
- Through grouping and sharing small quantities, pupils begin to understand division, and finding simple fractions of objects, numbers and quantities
- They make connections between arrays, number patterns, and counting in twos, fives and tens.

Year One Division

Key vocabulary:

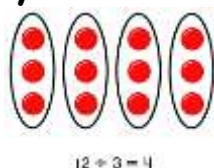
share, share equally, one each, two each..., group, groups of, lots of, array

Division Year Two

Group and share, using the \div and $=$ sign

Use objects, arrays, diagrams and pictorial representations, and grouping on a number line.

Arrays:

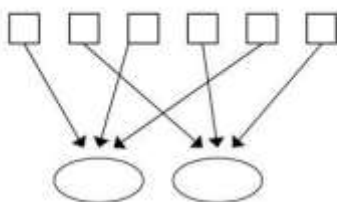


This represents $12 \div 3$, posed as how many groups of 3 are in 12?
Pupils should also show that the same array can represent $12 \div 4 = 3$ if grouped horizontally.

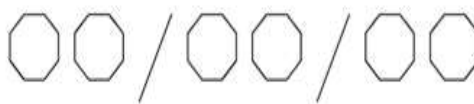
Know and understand sharing and grouping:

6 sweets shared between 2 people, how many do they each get?

Sharing



There are 6 sweets, how many people can have 2 sweets each?

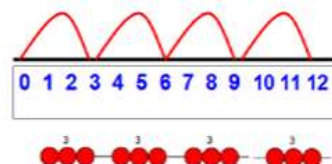


Grouping

Children should be taught to recognise whether problems require sharing or grouping.

Grouping using a number line:

Group from zero in equal jumps of the divisor to find out "how many groups of $_$ in $_$?". Pupils could use a bead string or practical apparatus to work out problems like "A CD costs £3. How many CDs can I buy with £12?" This is an important method to develop understanding of division as grouping.



$$12 \div 3 = 4$$

Key number skills needed for division at Year 2:

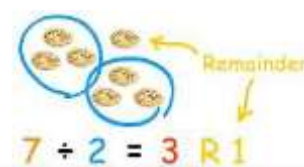
- Count in steps of 2, 3, and 5 from 0
- Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the \times , \div and $=$ signs
- Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

Year Two Division

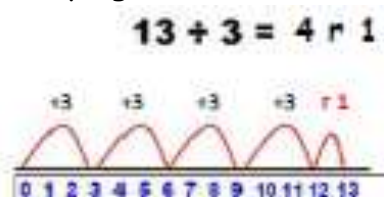
Key vocabulary: share, share equally, one each, two each..., group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over

Division Year Three

Divide 2-digit numbers by a single digit
(where there may be a remainder in the final answer)

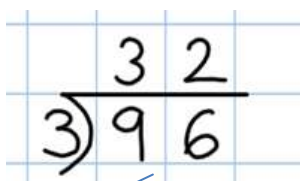


Grouping on a number line:

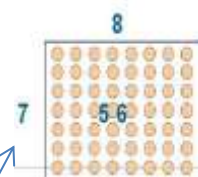


STEP 1: Children continue to work out unknown division facts by **grouping on a number line from zero**. They are also now taught the concept of **remainders**, as in the example. This should be introduced practically and with arrays, as well as being translated to a number line. Children should work towards calculating some basic division facts with remainders mentally for the 2s, 3s, 4s, 5s, 8s and 10s, ready for **carrying** remainders across within the short division method.

Short division: Limit numbers to **NO** remainders in the answer **OR** carried (each digit must be a multiple of the divisor).

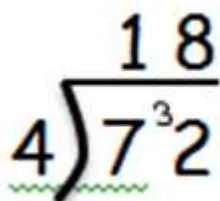


Demonstrate this using number lines, arrays etc., **short division** for larger 2-digit numbers should be introduced, initially with carefully selected examples requiring no calculating of remainders at all. Start by introducing the layout of short division by comparing it to an array.



Remind children of correct place value, that 96 is equal to 90 and 6, but in short division, pose: How many 3's in 9? = 3, and record it above the 9 tens.
How many 3's in 6? = 2, and record it above the 6 units.

Short division: Limit numbers to **NO** remainders in the final answer, but with remainders occurring within the calculation.



STEP 3: Once children demonstrate a full understanding of remainders, and also the short division method taught, they can be taught how to use the method when remainders occur within the calculation (e.g. $96 \div 4$), and be taught to **carry** the remainder onto the next digit. **If needed, children should use the number line to work out individual division facts that occur which they are not yet able to recall mentally.**

Real life contexts need to be used routinely to help pupils gain a full understanding, and the ability to recognise the place of division and how to apply it to problems.

Key number skills needed for division at Year 3:

- Recall and use multiplication and division facts for the 2, 3, 4, 5, 8 and 10 multiplication tables (through doubling, connect the 2, 4 and 8s)
- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- Solve problems, in contexts, and including missing number problems, involving multiplication and division
- Pupils develop efficient mental methods, for example, using multiplication and division facts (e.g. using $3 \times 2 = 6$, $6 \div 3 = 2$ and $2 = 6 \div 3$) to derive related facts ($30 \times 2 = 60$, so $60 \div 3 = 20$ and $20 = 60 \div 3$)
- Pupils develop reliable written methods for division, starting with calculations of 2-digit numbers by 1-digit numbers and progressing to the formal written method of short division.

Year Three Division

Key vocabulary: share, share equally, one each, two each..., group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, **inverse**, **short division**, **carry**, **remainder**, **multiple**

Division Year Four

Divide up to 3-digit numbers by a single digit (without remainders initially)

Continue to develop short division

Short division should only be taught once children have secured the skill of calculating „remainders“.

$$\begin{array}{r} 4 \overline{) 18} \\ \underline{16} \\ 2 \end{array}$$

STEP 1: Pupils must be secure with the process of short division for dividing 2-digit numbers by a single digit (**those that do not result in a final remainder** –see steps in Y3), but must understand how to calculate remainders, using this to carry remainders within the calculation process (see example).

$$\begin{array}{r} 54 \\ 4 \overline{) 218} \\ \underline{20} \\ 18 \\ \underline{16} \\ 2 \end{array}$$

STEP 2: Pupils move onto dividing numbers with up to **3-digits** by a single digit, however, problems and calculations provided should **not result in a final answer with remainder** at this stage. Children who exceed this expectation may progress to Year 5 level.

$$\begin{array}{r} 037 \\ 5 \overline{) 037} \\ \underline{00} \\ 37 \\ \underline{35} \\ 2 \end{array}$$

When the answer for the **first column** is zero ($1 \div 5$, as in example), children could initially write a zero above to acknowledge its place, and must always **carry** the number (1) over to the next digit as a remainder.

Real life contexts need to be used routinely to help pupils gain a full understanding, and the ability to recognise the place of division and how to apply it to problems. Include money and measure contexts when confident.

Key number skills needed for division at Year 4:

- Recall multiplication and division facts for all numbers up to 12×12
- Use place value, known and derived facts to multiply and divide mentally, including: multiplying and dividing by 10 and 100 and 1
- Pupils practise to become fluent in the formal written method of short division with exact answers when dividing by a one-digit number
- Pupils practise mental methods and extend this to three-digit numbers to derive facts, for example $200 \times 3 = 600$ so $600 \div 3 = 200$
- Pupils solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as three cakes shared equally between 10 children

Year Four Division

Key vocabulary: share, share equally, one each, two each..., group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple, divisible by, factor

Division Year Five

Divide up to 4 digits by a single digit, including those with remainders.

Short division, including remainder answers:

Short division with remainders: Now that pupils are introduced to examples that give rise to remainder answers, division needs to have a real life problem solving context, where **pupils consider the meaning of the remainder and how to express it**, i.e. as a fraction, a decimal, or as a rounded number or value, depending upon the context of the problem.

$$\begin{array}{r} 663 \text{ r } 5 \\ 8 \overline{) 5309} \end{array}$$

The answer to $5309 \div 8$ could be expressed as 663 and five eighths, $663 \text{ r } 5$, as a decimal, or rounded as appropriate to the problem involved.

See Year 6 for how to continue the short division to give a decimal answer for children who are confident.

Include money and measure contexts.

If children are confident and accurate:

Introduce **long division** for pupils who are ready to divide any number by a 2-digit number (e.g. $2678 \div 19$). This is a Year 6 expectation.

Key number skills needed for division at Year 5:

- Recall multiplication and division facts for all numbers up to 12×12 (as in Y4) Multiply and divide numbers mentally, drawing upon known facts
- Identify multiples and factors, including finding all factor pairs of a number, and common factors of two number
- Solve problems involving multiplication and division where larger numbers are decomposed into their factors
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000
- Use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- Work out whether a number up to 100 is prime, and recall prime numbers to 19
- Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- Use multiplication and division as inverses
- Interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding (e.g. $98 \div 4 = 24 \text{ r } 2 = 24 \frac{2}{4} = 24.5 \approx 25$)
- Solve problems involving combinations of all four operations, including understanding of the equals sign, and including division for scaling by different fractions and problems involving simple rates.

Year Five Division

Key vocabulary: share, share equally, one each, two each..., group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple, divisible by, factor, inverse, **quotient**, **prime number**, **prime factors**, **composite number (non-prime)**

Division Year Six

Divide at least 4 digits by both single-digit and 2-digit numbers
(including decimal numbers and quantities)

Short division, for dividing by a single digit: e.g. $6497 \div 8$

$$\begin{array}{r} 8 \overline{) 6497.000} \\ \underline{8} \\ 0 \\ \underline{8} \\ 0 \\ \underline{8} \\ 0 \\ \underline{8} \\ 0 \\ \underline{8} \\ 0 \end{array}$$

Short division with remainders: Pupils should continue to use this method, but with numbers to at least 4 digits, and understand how to express remainders as fractions, decimals, whole number remainders, or rounded numbers. Real life problem solving contexts need to be the starting point, where pupils have to consider the most appropriate way to express the remainder.

Calculating a decimal remainder: In this example, rather than expressing the remainder as r 1, a decimal point is added after the units because there is still a remainder, and the one remainder is carried onto zeros after the decimal point (to show there was no decimal value in the original number). Keep dividing to an appropriate degree of accuracy for the problem being solved. Pupils to be instructed to extend the division line as they add zeros.

Develop long division methods by chunking for dividing by 2 digits or more.

$$\begin{array}{r} 27 \\ 36 \overline{) 972} \\ \underline{- 720} \\ 252 \\ \underline{- 252} \\ 0 \end{array}$$

Answer : 27

Find out "How many 36s are in 972?" by subtracting **chunks** of 36, until zero is reached (or until there is a remainder).

Teach pupils to write a '**useful list**', first at the side that will help them decide what chunks to use.

Introduce the method in a simple way by limiting the choice of chunks to "Can we use 10 lots? Can we use 100 lots?" As children become confident with the process, encourage more efficient chunks to get to the answer more quickly (e.g. 20x, 5x), and expand on their '**useful**' lists.

Where **remainders** occur, pupils should express them as fractions, decimals or use rounding, depending upon the problem.

Key number skills needed for division at Y6:

- Recall and use multiplication and division facts for all numbers to 12×12 for more complex calculations
- Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context. Use short division where appropriate.
- Perform mental calculations, including with mixed operations and large numbers.
- Identify common factors, common multiples and prime numbers.
- Solve problems involving all 4 operations.
- Use estimation to check answers to calculations and determine accuracy, in the context of a problem.
- Use written division methods in cases where the answer has up to two decimal places.
- Solve problems which require answers to be rounded to specified degrees of accuracy.

Year Six Division

Key vocabulary: As previously, & common factor